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A Lens or Light Guide Using Convectively Distorted Thermal Gradients in Gases

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Development of systems for long-distance communications using laser beams is of considerable current interest. A major problem is to avoid losses in transmission systems for light. Losses by scattering and absorption in solid transmission schemes are likely to be large. By using gases as the media that guide the light beam, such losses should be minimized. A number of schemes in which gases can be made to guide a light beam have recently been investigated.

Tests on models of one type of gas lens or light guide utilizing refractive index gradients caused by temperature gradients are to be described in this paper. These particular models consisted of a suitably shaped continuous heating element or train of separate elements supported by insulating material inside a cooler pipe.

Heating elements composed either of a series of closely spaced, doughnut-shaped rings or toruses, or of a single, continuous helix of suitable dimensions were used in the experiments to be described. The beam travels through the doughnut holes or down the axis of the helix. Helices were used in the more detailed tests to be reported here because they are much easier to construct and to support in the cooler outer jacket. They focus at least as strongly and with as little aberration as toruses of approximately equivalent dimensions.

Lens trains of the types described focus to some extent by the alternating gradient (AG) focusing mechanism.^{*1,2} However, if the temperature distribution is suitably distorted by moving the gas past the heating element, a great enhancement of focusing can be obtained. The enhancement is due to the ordinary focusing effect of a radially decreasing average refractive index, which is absent if the gas is static. Convection alone caused the gas motion in the lenses described here.

The best of several experimental models of convective thermal gas

* A. R. Hutson first proposed use of AG focusing in gas lenses to the author in private discussion.

lenses, in terms of specific convergence* vs power consumption by the heating element as well as absence of aberrations, is illustrated in Fig. 1. The model contains a helix about 0.75 meters long and is closed at the ends with flat glass plates to confine the gas. In tests of power consumption the helix was warmed electrically, but for detailed tests for convergence vs temperature difference between helix and outer jacket, it proved more convenient to run warm water through the helix, which was made of copper tubing. Temperature differences were measured with a thermocouple (not shown), fastened to the helix and to the inside of the outer jacket. Measurements of focusing strength and aberration were made with considerable precision using a modification of the Foucault knife-edge test.³ A very fine pinhole light source, two 80-cm telescope objective lenses (one for making the incident light parallel and the other for enhancing its reconvergence), and both horizontal and vertical knife edges were used in the tests. The two knife-edge orientations enabled us to measure not only focal length, but also spherical aberration, astigmatism, and a type of aberration that might be characterized as "S-shaped" or "sagging lens" aberration. The latter was revealed by the *horizontal* knife-edge test as an inequality in focal length between the upper and lower halves of the clear aperture. One should expect astigmatism and sagging lens aberration in a convective system.

The amount of astigmatism is surprisingly small, and the amounts of spherical and "sagging lens" aberration were too small to detect with the gas lens shown in Fig. 1, using air, CO₂ or propane gas at temperature differences that gave focal lengths of more than about 5 meters or specific convergences less than about 0.3 meters⁻². At higher temperature differences, both astigmatism and sagging lens aberration began to appear, but were relatively small perturbations on the convergence. Fig. 2 shows a detailed study of the astigmatism and convergence ($1/F$) versus temperature difference (helix temperature minus jacket temperature) with the jacket at approximately room temperature using CO₂ at one atmosphere pressure.

When helices with considerably larger apertures for the light beam

* The periodic light guides described here had the property that the focal length of any one cycle of the guide was very much longer than the length of one cycle. Consequently, AG focusing was very weak and the focusing was due largely to the fact that the mean refractive index along the axis was higher than along parallel lines off the axis. The mean refractive index at radius r from the axis can be described by the equation $n = n_0(1 - \frac{1}{2}Cr^2)$ when aberrations are absent. When AG focusing is negligible, the quantity C is equal to average focusing power, or convergence, of a segment of unit length out of the continuous "lens train." Hence, it is appropriate to call C the *specific convergence* of such a light guide, a term suggested by W. L. Bond. The maximum allowable curvature of such a light guide is proportional to its specific convergence and to the diameter of its aperture.

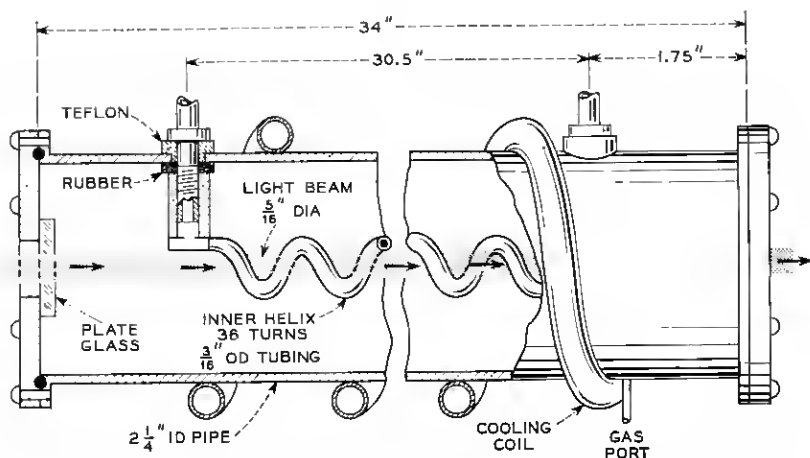


Fig. 1 — Convective gas lens showing the best of several warming elements, a helix, inserted in such a way as to be heated with warm water; the jacket is also kept at a fixed lower temperature with water.

were tested, there was a large amount of spherical aberration when pure CO_2 was used, but not when a mixture of half CO_2 and half helium or when pure argon was used. Likewise, the helix shown in Fig. 1 gave measurable spherical aberration when C_4F_{10} was used but not with any of the lighter gases tested. A helix with an even smaller aperture would probably greatly reduce the spherical aberration with C_4F_{10} . These facts are consequences of the differences in relative rates of convective and conductive heat flow in the gases.

Another effect that depends on these factors, and also on the specific refractive index of the gases, is the specific convergence when the power per unit length consumed by the heating element is fixed, or what may be called the *efficiency* of the lens. Efficiency has the dimensions of $\text{meters}^{-1} \text{watts}^{-1}$, or *dipters per watt*. The efficiency of the lens shown in Fig. 1 was found to be almost independent of power consumption in the range where aberrations were small. Measurements at different pressures with CO_2 showed that efficiency is approximately proportional to pressure. When one watt of electrical power was supplied continuously to the helix with the jacket at room temperature, the temperature differences, efficiencies and values of specific convergence shown in Table I were measured with the gases listed.

The amount of AG focusing with a helical heating element was calculated and shown to be entirely negligible when the helix turns are as closely spaced as in the lens shown in Fig. 1. This is confirmed by the

TABLE I

Gas	ΔT	Efficiency	Specific Convergence
air	4.0°C	0.045 m ⁻¹ w ⁻¹	0.06 m ⁻²
CO ₂	3.9°C	0.091 m ⁻¹ w ⁻¹	0.12 m ⁻²
C ₃ H ₈	3.4°C	0.12 m ⁻¹ w ⁻¹	0.16 m ⁻²
C ₄ F ₁₀ *	2.6°C	0.15 m ⁻¹ w ⁻¹	0.20 m ⁻²

* This gas was suggested by K. B. McAfee. The focal length is only approximate because of spherical aberration.

symmetry of Fig. 2, since AG focusing would be independent of the sign of the temperature difference. Fig. 3(a) is a three-dimensional plot showing how the refractive index, n , would vary as a function of position in a plane normal to the optic axis if there were negligible convection. In that case, both the temperature and the refractive index would very nearly obey Laplace's equation. Suppose one considers two orthogonal cross sections of the plot parallel to and containing the optic axis. Near the axis, a cross section that cuts through the helix has curvature $\partial^2 n / \partial y^2$ that is equal in magnitude but opposite in sign to the curvature $\partial^2 n / \partial x^2$ in the orthogonal cross section. (Cf. lines YOY' and XOX' on Fig. 3a.) This is a direct consequence of the Laplace equation and the symmetry

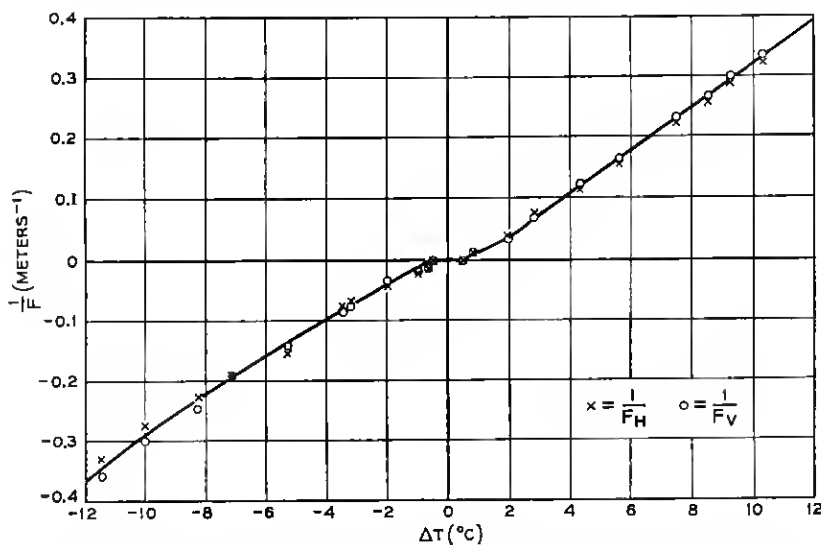


Fig. 2 — Reciprocal focal length, or convergence in vertical plane ($1/F_V$) and in horizontal plane ($1/F_H$), vs temperature of inner helix minus temperature of outer tube for the lens shown in Fig. 1 with CO₂ at one atmosphere pressure. The difference between horizontal and vertical convergence is a measure of astigmatism.

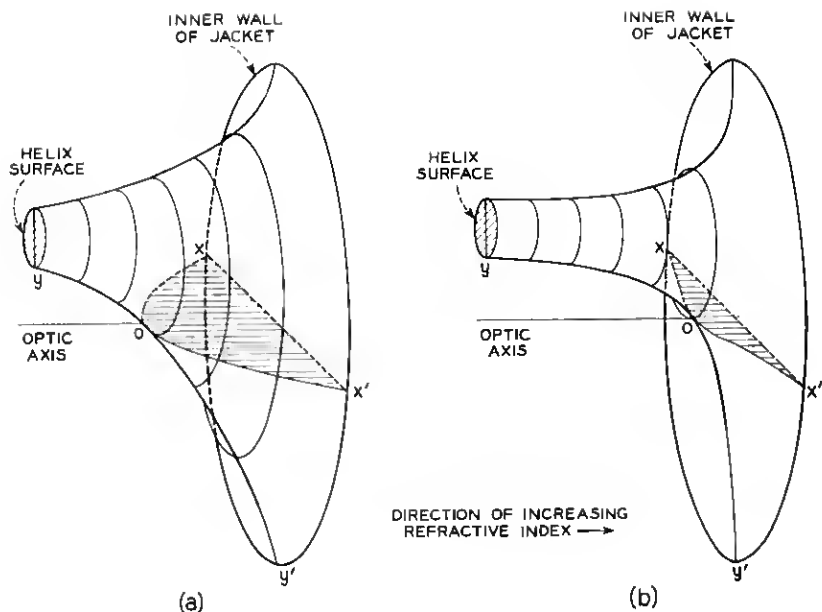


Fig. 3 —(a) Refractive index vs two coordinates of position in a plane normal to the optic axis in the gas lens. The refractive index is highest at the jacket wall, represented by the large circle seen in perspective on the right side, and lowest at the warm helix wall, represented in cross section by the small circle on the left. This figure represents the hypothetical situation in the absence of convection. (b) Similar to (a) but with convection. Note difference in magnitudes of curvature of the plot in cross sections through the optic axis normal (XOX') and parallel (YOY') to the plane of the drawing. Such difference is zero in (a). Details would vary somewhat with azimuth of helix at the plane of the plot.

of the helix. No ordinary focusing occurs in this case, because there is no change in *average* refractive index with radial distance from the axis on lines parallel to the axis. A more rigorous analysis shows that no ordinary focusing occurs anywhere inside the aperture of any system having either helical or ordinary axial symmetry if n obeys Laplace's equation. Fig. 3(b) is the same sort of plot as Fig. 3(a), except that the contours are changed by convection. The convection produces a sharper temperature gradient near the helix surface and the temperature is more nearly uniform elsewhere. Consequently, the plot of refractive index curves more sharply, near the axis, than before in the cross section through the axis and the helical tube, and less sharply in the orthogonal direction. The result is that the *average* refractive index is then higher along lines near the axis than along lines farther away, so that there is a net positive ordinary focusing effect.

A detailed mathematical analysis of such convective flow has not been

made to the author's knowledge, but experiments on temperatures and flow in gases between concentric cylinders of unequal temperature have been reported in Ref. 4. That paper shows temperature distribution alteration by convection in a quantitative way and also presents the relation between convection and the relative size of inner and outer cylinders and the viscosity, specific heat, density and heat conductivity of the convective fluid.

Although any element of the helix is far from straight and is not very near the center in any cross-sectional element, the convective flow in any plane is qualitatively very similar to that described for small flow rates in Ref. 4. The flow patterns in several planes were observed and photographed in a glass-walled replica of the lens using tobacco smoke in air. The flow was perfectly steady and nonturbulent, even with a temperature

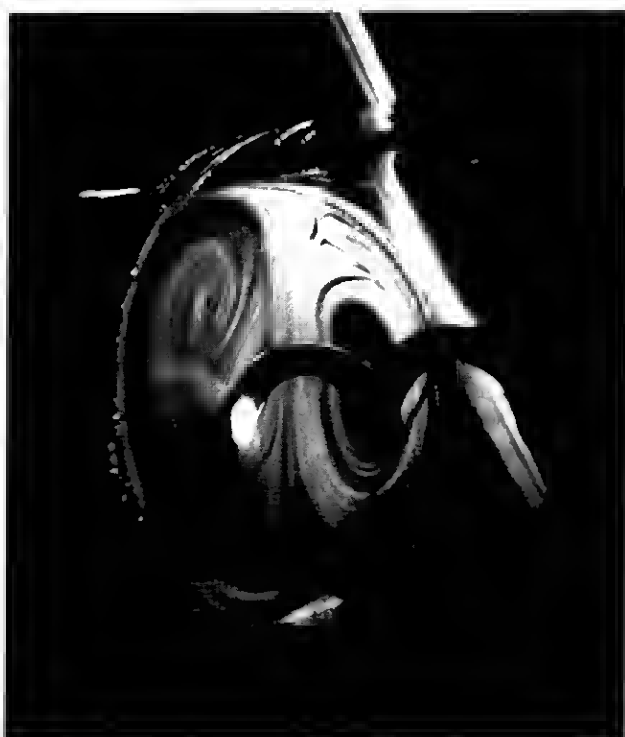


Fig. 4 — Smoke moving with air in glass-walled gas lens. The camera is somewhat below and to one side of the lens. Smoke is flowing upward around the helix and downward on both sides, inside the jacket. Helix is 10°C warmer than the jacket. Illumination is from the upper right through a narrow slit normal to the axis. The illuminated region cuts through the helix at a point level with the axis.

difference of over 10°C , and always showed a two-kidney shaped pattern similar to that obtained with a concentric cylinders.⁴ Fig. 4 is one of these photographs, taken in a plane where the helical tube is on a horizontal line from the axis, where one might expect greatest departure from the results with concentric cylinders.

It is evident, especially from Ref. 4, that temperature gradients are not axially symmetric even when averaged over a complete loop of the helix, because of the relation between the convective flow patterns and the direction of gravity. It is therefore surprising, but none the less true, that nearly perfect, aberration-free focusing can be achieved with the convective gas lens if a uniform helix of the proper cross-sectional dimensions is chosen for the warming element. There seems to be no reason why a continuous helix and tubular jacket of similar cross section but of great length might not have the same local focusing properties as the short segment that was tested.

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